JOURNAL OF MATERIALS SCIENCE 30 (1995) 3483–3486

Void behaviour due to internal pressure induced by temperature rise

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In this paper, the governing equations for the ductile growth of an isolated void in a nonhardening material, due to the internal pressure induced by temperature rise, are derived. It is found that the enlargement of the void volume is enhanced by the elevation of the temperature. In the case of the temperature rise caused by the plastic dissipation, the void behaviour considering temperature effect can be determined by combining with heat conduction equation.

1. Introduction

It has been frequently observed that the temperature at the tip of a propagating crack can give an apparent rise [see 1–3], which has a great influence on the fracture mechanisms [1–3]. The presence of high temperatures in the plastically deformed region at the tip of running cracks in polymethylmethacrylate (PMMA) and polystyrene (PS) has been confirmed experimentally by Fuller *et al.* [1], and it has been found that for such kind of materials, the monomer evolved could aid the formation of the voids which occupy 40% of the volume of the crazed material. An additional factor to be considered is that the material will contain some water (e.g. about 0.6% by mass in PMMA), which if vaporized would generate a void volume comparable to that formed by the monomer evolved. Generally within the voids the relationship between the internal pressure p and temperature T can be expressed as

$$g(p, V, T) = 0 \qquad (1)$$

where V is the void volume. For the ideal gas the following relation holds

$$\frac{pV}{T} = \text{const.} \qquad (2)$$

It is necessary to examine the effect of temperature rise. McClintock [4] has presented a start on the void problem through his analysis of the expansion of a long circular cavity in a non-hardening material, pulled in the direction of its axis while subjected to transverse tensile stresses. Rice and Tracey [5] considered a spherical void in an infinite matrix, and found an exponential dependence of void growth rate on triaxial stress. Gurson [6] developed a method for calculating approximate yield loci via an upper bound approach for porous ductile materials.

Our present work seeks to determine the relation between the void growth and temperature rise. We start by deriving a variational principle governing cavity expansion in an infinite rigid–plastic medium, which considers the tractions applied on the cavity induced by temperature rise. Then the governing equations for void growth via the temperature are derived, both for a long cylindrical void and a spherical void. Numerical results and discussions are presented.

2. Variational principle for void growth in rigid–plastic materials and subjected to internal pressure

Consider an infinite body of an incompressible rigid–plastic material, containing an internal void or voids with boundary surface S_v, on which tractions $p_i (i = 1, 2, 3)$ are exerted. At the current instant the material is subjected to a uniform remote strain rate field $\dot{\varepsilon}_{ij}^{\infty}$. This determines the remote deviatoric stress state s_{ij}^{∞} and, in addition, the current remote mean normal stress σ^{∞} is specified so that

$$\sigma_{ij}^{\infty} = s_{ij}^{\infty} + \sigma^{\infty}\delta_{ij} \qquad (3)$$

An extension for the functional to this problem can be followed from Budiansky and Vidensek's variational method [7]. Now define a functional $Q(\dot{\mathbf{u}})$ of any velocity field $\dot{\mathbf{u}}$ as

$$Q(\dot{\mathbf{u}}) = \int_V [s_{ij}(\dot{\varepsilon}) - s_{ij}^{\infty}]\dot{\varepsilon}_{ij}\mathrm{d}V - \int_{S_v} (\sigma_{ij}^{\infty} n_i + p_i)\dot{u}_j \mathrm{d}S \qquad (4)$$

where V denotes the infinite volume exterior to the void(s) and, n_i is a unit normal drawn into the material. The following convergence assumptions should be

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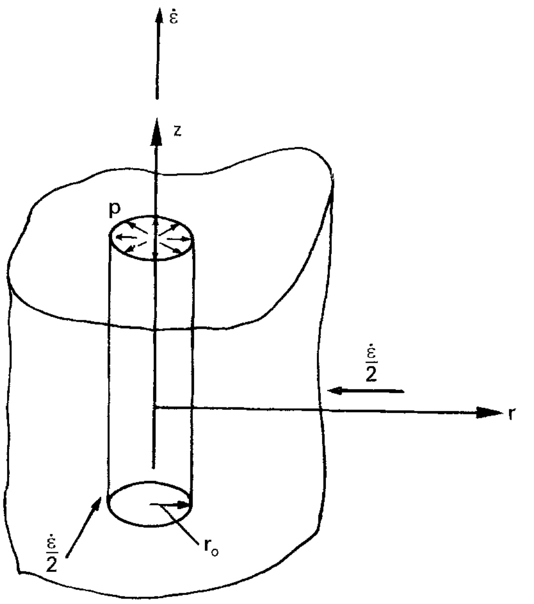

Figure 1 A cylindrical void with internal pressure.

satisfied

$$\lim_{S_e \to \infty} \int_{S_e} (\sigma_{ij}^A - \sigma_{ij}^\infty) n_i (\dot{u}_j^2 - \dot{u}_j^1) dS = 0 \quad (5)$$

and

$$\int_V [s_{ij}(\dot{\varepsilon}^1) - s_{ij}^\infty](\dot{\varepsilon}_{ij}^2 - \dot{\varepsilon}_{ij}^\infty) dV \quad (6)$$

is bounded. The subscripts 1 and 2 refer to any two of these fields and superscript A denotes the actual field. Considering any velocity field $\dot{u}_j$ satisfying the incompressibility and agreeing with the remote strain rate, i.e.

$$\dot{\varepsilon}_{ij} = \frac{1}{2}(\dot{u}_{i,j} + \dot{u}_{j,i}) \to \dot{\varepsilon}_{ij}^\infty \quad x_i x_i \to 0; \quad \dot{\varepsilon}_{ii} = 0 \quad (7)$$

And then it can be proved for any velocity field $\dot{u}_j$

$$Q(\dot{\mathbf{u}}) - Q^A = \int_V [s_{ij}(\dot{\varepsilon}) - s_{ij}^A]\dot{\varepsilon}_{ij} dV > 0 \quad (8)$$

We will employ the minimum principle as a basis for the approximate solutions via the Rayleigh–Ritz method in the following analysis.

3. Growth of a cylindrical void with internal pressure induced by temperature rise

A long cylindrical void (Fig. 1) stretched at a uniform rate $\dot{\varepsilon}$ in the direction of its axis while subjected to internal pressure p, is considered. A remote transverse stress σ_{rr}^∞ is also specified. In the view of axial symmetry and incompressibility, the admissible velocity field is

$$\begin{cases} \dot{u}_r = \left(\dot{r}_0 + \frac{1}{2}\dot{\varepsilon} r_0\right)\frac{r_0}{r} - \frac{1}{2}\dot{\varepsilon} r \\ \dot{u}_z = \dot{\varepsilon} z \\ \dot{u}_\theta = 0 \end{cases} \quad (9)$$

where $\dot{r}_0$ is the unknown transverse velocity of the cavity boundary. Substituting Equation 9 into the functional Equation 4, one gets

$$Q(\dot{\mathbf{u}}) = \int_{r_0}^\infty [s_{ij}(\dot{\varepsilon}) - s_{ij}^\infty]\dot{\varepsilon}_{ij} 2\pi r dr - (\sigma_{rr}^\infty + p)\dot{r}_0 2\pi r_0 \quad (10)$$

where

$$s_{ij}(\dot{\varepsilon}) = \sqrt{2}\tau_0 \dot{\varepsilon}_{ij}/(\dot{\varepsilon}_{kl}\dot{\varepsilon}_{kl})^{1/2} \quad (11)$$

in the case of a non-hardening Mises material with yield stress $\tau_0(T)$ in shear, where τ_0 generally is the function of temperature T.

Since the actual velocity field has the form of Equation 9, we get the exact answer by minimizing Q with respect to $\dot{r}_0$, which leads to

$$\frac{\dot{r}_0}{r_0} = \frac{3^{1/2}}{2}|\dot{\varepsilon}| \sinh\left[\frac{\sigma_{rr}^\infty + p(T, V_r)}{\tau_0(T)}\right] - \frac{1}{2}\dot{\varepsilon} \quad (12)$$

where V_r is the cavity volume and can be expressed in term of $r_0(t)$

$$V_r = \pi r_0^2(t) L \quad (13)$$

and L is the length of the void. Based on Equations 2 and 13, Equation 12 then reads

$$\frac{\dot{r}_0}{r_0} = \frac{3^{1/2}}{2}|\dot{\varepsilon}| \sinh\left[\frac{\sigma_{rr}^\infty}{\tau_0(T)} + \frac{T R_0^2 p_0}{T_0 r_0^2 \tau_0(T)}\right] - \frac{1}{2}\dot{\varepsilon} \quad (14)$$

where R_0 is the initial radius of void, i.e. $R_0 = r_0(0)$ and p_0 is the initial pressure.

4. Growth of a spherical void via temperature rise

Consider a single spherical void subjected to internal pressure in a general remote strain rate field $\dot{\varepsilon}_{ij}^\infty$, with the remote normal stress σ^∞ being specified (see Fig. 2). According to Rice and Tracey [5], any assumed velocity fields can be expressed as

$$\dot{u}_i = \dot{\varepsilon}_{ij}^\infty x_j + D\dot{u}_i^D + E\dot{u}_i^E \quad (15)$$

where the first term on the right-hand side relates to a velocity in a uniform strain rate field $\dot{\varepsilon}_{ij}^\infty$, and D and E are assumed to be determined. $\dot{u}_i^D$ is a spherically symmetric volume changing field, and $\dot{u}_i^E$ is a shape changing field which preserves void volume. Incompressibility and spherical symmetry require that the volume changing field be

$$\dot{u}_i^D = \left(\frac{2}{3}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}\right)^{1/2}\left(\frac{r_0}{r}\right)^3 x_i \quad (16)$$

It has been found that for a tensile remote field the spherically symmetric volume changing part of the void growth far overwhelms the shape changing part when the remote mean stress is large. It is possible to choose an assumed velocity field involving only the contribution from the remote strain field and a spherically symmetric void expression field, i.e.

$$\dot{u}_i = \dot{\varepsilon}_{ij}^\infty x_j + D\dot{u}_i^D \quad (17)$$

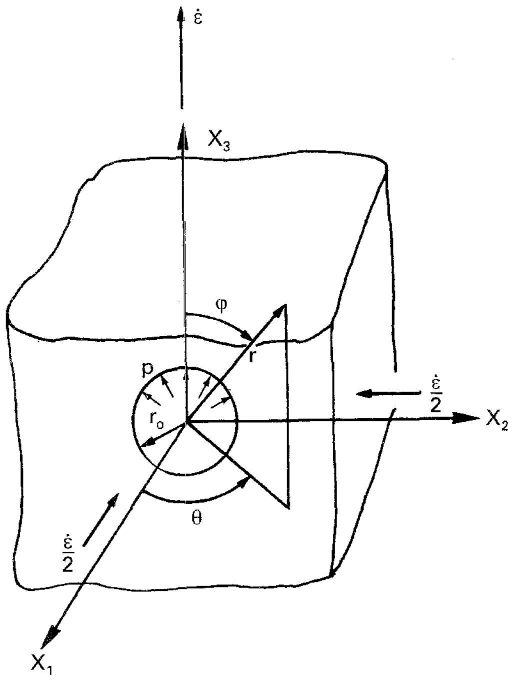

Figure 2 A spherical void with internal pressure.

By substituting Equation 17 into Equation 4 and minimizing the functional Q, we get

$$\int_{S_v} [s_{ij}(D) - s_{ij}^{\infty}]\dot{\varepsilon}_{ij}^{D}\mathrm{d}V = (\sigma^{\infty} + p)\int_{S_v} n_i \dot{u}_j \mathrm{d}S \tag{18}$$

where for a nonhardening Mises material, the deviatoric stresses corresponding to the assumed field are given by Equation 11 as

$$s_{ij}(D) = \frac{\sqrt{2}\tau_0(\dot{\varepsilon}_{ij}^{\infty} + D\dot{\varepsilon}_{ij}^{D})}{(\dot{\varepsilon}_{ij}^{\infty}\dot{\varepsilon}_{ij}^{\infty} + 2D\dot{\varepsilon}_{ij}^{\infty}\dot{\varepsilon}_{ij}^{D} + D^2\dot{\varepsilon}_{ij}^{D}\dot{\varepsilon}_{ij}^{D})^{1/2}} \tag{19}$$

Following the similar analysis to [5], after complicated but straightforward calculations, finally one gets

$$\frac{\dot{r}_0}{r_0}(\tfrac{2}{3}\,\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij})^{-1/2} = C(\nu)\exp\left[\frac{3^{1/2}}{2}\left(\frac{\sigma_{rr}^{\infty}}{\tau_0(T)} + \frac{TR_0^2 p_0}{T_0 r_0^2 \tau_0(T)}\right)\right] \tag{20}$$

where $C(\nu)$ is the constant dependent on the strain rate component, and can be approximately expressed as

$$C(\nu) = 0.279 + 0.004\nu \tag{21}$$

where

$$\nu = \frac{-3\dot{\varepsilon}_{II}^{\infty}}{\dot{\varepsilon}_{I}^{\infty} - \dot{\varepsilon}_{III}^{\infty}} \tag{22}$$

For a simple tension remote field, $C(\nu) = 0.283$, which will be used in following analysis.

5. Results and discussions

Generally, it should be noted that, from Equations 13 and 20, the void growth will be dependent on the history of the temperature rise, since the temperature should be regarded as the function of time t. For simplicity, here we first examine the initial void growth rate $\dot{r}_0/r_0(2/3\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij})^{-1/2}$ via temperature rise, in different applied mean stress levels, as plotted in Fig. 3 for a cylindrical void and in Fig. 4 for a spherical void. In both cases, the initial dimensionless pressure p_0/τ_0 is taken as 0.1 where τ_0 is assumed to be constant. It can be seen that the initial void growth rate increases with the temperature increases. It is interesting to note that the variation of $\dot{r}_0/r_0(2/3\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij})^{-1/2}$ with temperature is almost linear both for cylindrical and spherical voids, in a wide range of applied mean stresses.

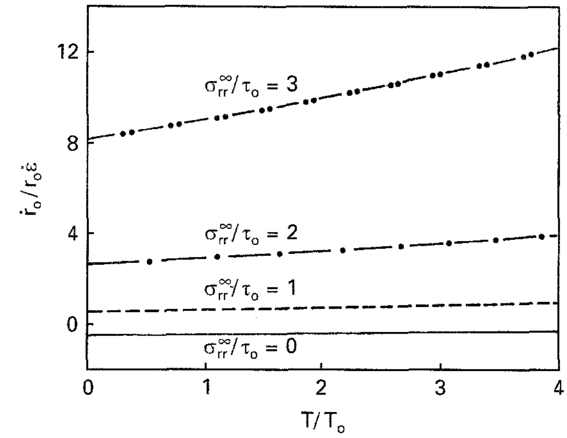

Figure 3 The initial cylindrical void growth rate $\dot{r}_0/r_0\dot{\varepsilon}$ via temperature rise. –··– $\sigma_{rr}^{\infty}/\tau_0 = 3$; –·– $\sigma_{rr}^{\infty}/\tau_0 = 2$; ––– $\sigma_{rr}^{\infty}/\tau_0 = 1$; — $\sigma_{rr}^{\infty}/\tau_0 = 0$.

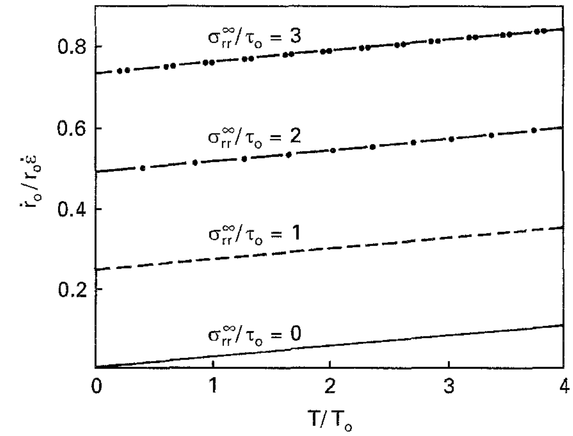

Figure 4 The initial spherical void growth rate $\dot{r}_0/r_0\dot{\varepsilon}$ via temperature rise. –··– $\sigma_{rr}^{\infty}/\tau_0 = 3$; –·– $\sigma_{rr}^{\infty}/\tau_0 = 2$; ––– $\sigma_{rr}^{\infty}/\tau_0 = 1$; — $\sigma_{rr}^{\infty}/\tau_0 = 0$.

Figs 5 and 6 give the results of the variation of the dimensionless void radii r_0/R_0 as a function of the change of remote strain $\Delta\varepsilon$, at different temperatures, for a cylindrical void and spherical void respectively. The results with solid lines correspond to the temperature effect and are not taken into account. It can be found that the temperature rise has great influences on the void enlargement. Because in reality the yield stress will generally decrease with the temperature increase, the results presented here assuming τ_0 to be constant, are more conservative.

For a running crack problem, the temperature rise at the crack tip is induced by the plastic dissipation. According to the Fourier heat conduction law, the governing equation for temperature rise is

$$\frac{\partial T}{\partial t} - \nabla^2 T - \sigma_{ij}\dot{\varepsilon}_{ij} = 0 \tag{23}$$

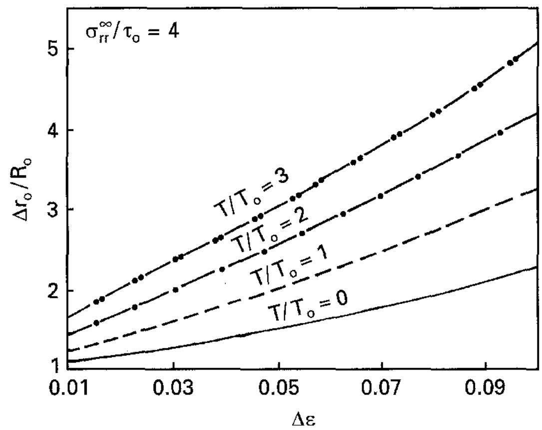

Figure 5 r_0/R_0 as the function of $\Delta\varepsilon$ (for cylindrical void). $\sigma_{rr}^{\infty}/\tau_0 = 4$; –··– $T/T_0 = 3$; –·– $T/T_0 = 2$; --- $T/T_0 = 1$; — $T/T_0 = 0$.

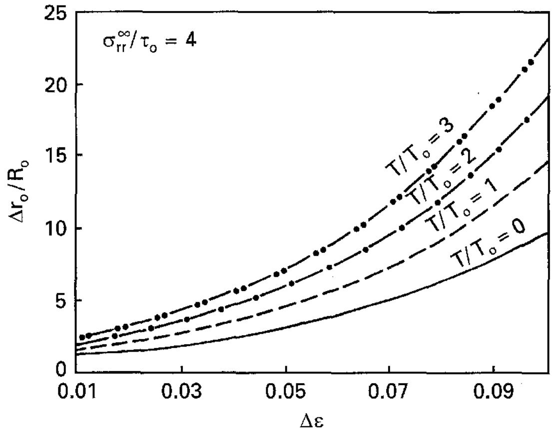

Figure 6 r_0/R_0 as the function of $\Delta\varepsilon$ (for spherical void). $\sigma_{rr}^{\infty}/\tau_0 = 4$; –··– $T/T_0 = 3$; –·– $T/T_0 = 2$; --- $T/T_0 = 1$; — $T/T_0 = 0$.

The temperature rise as a function of t can be solved from the above equation once the plastic deformation field is known, and then, combined with Equations 14 or 20, the void behaviour can be totally determined. Moreover, in view of the effects of the interaction along the voids, it might be possible to extend Gurson's model [6] for such problems, and obtain the related yield criteria and constitutive equation, in which the temperature effect is considered. This work will be continued further.

Acknowledgement
This work is supported by Japan Society for Promotion of Science and, National Natural Science Foundation of China. The author is very grateful for discussions with K. Watanabe, Institute of Industrial Science, University of Tokyo.

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Received 21 December 1993
and accepted 25 November 1994